Development of phased strong-motion time-histories for structures with multiple supports

Bruce A. Bolt[1]

ABSTRACT

Complete seismic analysis of critical structures, particularly with long spans such as bridges and elevated viaducts, requires realistic predictions of free-field ground motions at all the interface points on the supporting foundation under design earthquake conditions. Time histories differ slightly in wave form at sequential points because of wave propagation differential velocities, irregularities in rock and soil structure, and wave emission delays at the fault rupture. The effects in the input motions show up as phase shifts in each frequency component of the P, S and surface seismic waves. The result can be measured in terms of coherency factor which is a function of both wave frequency and support separation.

A dynamic response ratio has been defined for a discrete linear structural system in order to allow for the above spatial variations of the ground motions. It has been shown using the SMART 1 array recordings that structural response with differential phasing may be reduced by 25 percent from non-phased input response at 5 Hz for spans of 200 m.

A procedure for the synthesis of sets of phase time-histories is outlined using both time domain and seismic phase response spectral methods. Examples from recent work on phased input for long bridges in the San Francisco Bay are given.

INTRODUCTION

Large structures with a rigid foundation or multiple supports tend to respond

1 Professor of Seismology, Department of Geology and Geophysics and Civil Engineering, University of California at Berkeley, CA 94720.

(Luco and Wong, 1986) so as to average the free-field accelerations incident upon the supports (Figure 1). The structural dynamic analysis can be either by suitably phased ("lagged") time histories applied in an appropriate way at each support or by a modal response analysis complete with phase information. The usual response spectrum, as defined by Housner, describes the amplitude of the oscillator motion but does not include information on its phase behavior. However, for large structures, given the observed seismic ambient frequencies, it is necessary to also consider wave phase because wave offsets over inter-support distances produce differential ground acceleration along the base of the structure. The same argument holds for differential rotations.

In all cases, non-vertically propagating seismic waves produce systematic phase shifts between support points, producing significant incoherency which is reinforced by incoherencies produced in other ways.

As data from dense strong motion arrays with absolute times become available, there will be more opportunities to study phase changes and coherency in seismic ground motions over various distances and to estimate this effect on large engineered structures. In the modal analysis option, to facilitate the analysis of the phasing, a definition of a response phase spectrum has been developed (Abrahamson and Bolt, 1985), which is consistent with the response (amplitude) spectrum currently used by engineers. This method allows the routine calculation of the response phase spectrum. The examples given refer to ground acceleration, but the defined response phase is also valid for ground velocity and displacement.

One use of a response phase spectrum is demonstrated by developing a simple method for estimating the dynamic response ratio for large structures using only the response amplitude and phase spectra. The method greatly simplifies the estimation of the response ratio for multiple excitation input points.

The introduction of ground motion phasing in engineering practice is at the present time in its infancy and is probably necessary only in critical cases. At this stage the main approach is largely an empirical one with simple adjustments in onset times to observed strong motion records. The justification of the incorporation of phase shifts, is firstly, that a reduction in response is normally to be expected relative to in-phase motion and secondly, non-linear effects are significant.

WAVE PHASING DEFINED

Time Domain and Spectral Domain

Earthquake ground shaking at a point (or station) on the ground surface consists of a mixture of different elastic wave types with time as the independent variable.

The motion ("time history") is rarely stationary in time but varies in amplitude A, frequency f ($2\pi f = \omega$ radians per sec) and wavelength λ from time window to time window. Nevertheless, the motion can be represented by the superposition of simple harmonic (advancing) plane waves. The wave number k is then a useful concept, where

$$k = 2\pi/\lambda. \tag{1}$$

Linear strong motion array or multiple support analysis can be expressed as filtering followed by a summation. A standard assumption in array analysis is that the record consists of a deterministic signal plus noise. For example,

$$u_j(t) = u(x_j, t) = s(t) + \varepsilon(x_j, t), \tag{2}$$

where $u_j(t)$ is the output (acceleration, velocity, or displacement) of the jth seismometer at position x_j, s(t) is the deterministic signal, and $\varepsilon(x_j, t)$ is the noise. The form of Eq. (2) assumes that the signal arrives simultaneously at each station. This condition can be satisfied by introducing a delay τ_j to the output of the jth seismometer. For a plane wave, the delay is

$$\tau_j = \mathbf{k} \bullet x_j/\omega, \tag{3}$$

where $\mathbf{k}$ is the vector wave number and ω is the frequency of the plane wave.

In some analyses it is sufficient to consider just a single harmonic component of the strong motion record

$$s(x, t) = A \cos(\mathbf{k} \bullet \mathbf{x} - \omega t + \delta), \tag{4}$$

where A and δ are constants. The wave velocity is $c = \omega/|\mathbf{k}|$.

In Eq. (4), the angle δ is called the phase angle and, since $S = A \cos \delta$ when $x = 0$, $t = 0$, it clearly represents an advance (or delay) of the whole harmonic with respect to the spatial or temporal origin (see Figure 2). In this discussion we are interested particularly in the phase angle δ.

An alternative representation to that in the time domain given by Eq. (4) above is to transform the motion to an equivalent description in which frequency is the independent variable. This change is usually accomplished mathematically by computing the Fourier transform of u(t) which we could write as $\overline{u}(\omega)$ or simply $u(\omega)$. A similar transformation can be made from location space x to wave number space k. For multiply-supported structures or free-field arrays, the input motions u(x, t) represent a time-dependent, three-dimensional wave field. The wave field may be written as a three-dimensional (generalized) Fourier transform

$$u(x,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u(k,\omega) \exp\{-i(k\cdot x - \omega t)\}\, dk\, d\omega, \tag{5}$$

where k is the wave number vector and ω the frequency. The inverse transform is

$$u(k,\omega) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u(x,t) \exp\{i(k\cdot x - \omega t)\}\, dx\, dt. \tag{6}$$

The amplitude-squared ground motion spectrum is given by

$$f_{uu}(k,\omega) = |u(k,\omega)|^2. \tag{7}$$

To calculate u(k, ω), the spatial integral in Eq. (6) is replaced by a weighted sum over the sampled station distribution. The weights W_j correspond to the filters adopted in the numerical program. The spectral estimate in Eq. (7) is then

$$f_{uu}(k,\omega) = \left| \sum_{j=1}^{N} W_j\, u(x_j,\omega) \exp\{ik\cdot x\} \right|^2 \tag{8}$$

$$= \sum_{j=1}^{N}\sum_{l=1}^{N} W_j\, W_l\, u(x_j,\omega)\, \overline{u}(x_l,\omega) \exp\{ik\cdot (x_j - x_l)\}$$

$$= W\, \overline{W^T}\, \overline{U^T}(k)\, S(\omega)\, U(k). \tag{9}$$

In Eq. (9) the factor S(ω) is called the cross-spectral matrix and as we see plays the most important role in measuring the coherency of the strong ground motion across the site.

It is crucial to note that a Fourier transform is complex (a + ib, say). It has both a modulus ($\sqrt{a^2 + b^2}$) and an argument (arctan b/a). Therefore, the complete representation of a time history Eq. (2) is an amplitude spectrum plus a phase spectrum ("a spectrum pair"). In engineering applications with a rigid base or single input point, the response phase spectrum is normally unimportant and is ignored. When the phase of strong-motion time histories becomes a factor, then the complete response spectrum pair must be considered.

It should be noted that the author suggested two decades ago generating new and more suitable time-histories from available accelerograms by cross-over of phase spectra. This substitution procedure preserves the peak motions and spectral power but with a different duration and phasing pattern.

Seismic Wave Considerations

The construction of seismologically realistic time histories (synthetic seismograms) requires that the different types of seismic waves and their interrelationships be correctly incorporated (see Bullen and Bolt, 1985 for a full description). First, seismic waves, like light waves, can be polarized so that, for example, the vertical component of motion may be quite different from either the radial horizontal component (i.e. in the direction from source to station) or the transverse horizontal component.

We need to consider in most applications only three types of seismic waves: compressional (P) waves, shear (S) waves and surface (Love and Rayleigh) waves. The velocities of these waves depend upon the elastic moduli of the rocks and soil through which they propagate. We have

$$v_P = \sqrt{\frac{k + \frac{4}{3}\mu}{\rho}}, \qquad v_S = \sqrt{\frac{\mu}{\rho}}, \qquad (10)$$

where k is the bulk modulus, μ the rigidity and ρ the density. It follows that $v_P > v_S$ always. The speed of surface waves is always less than or equal to v_S. S waves do not travel in water and all types of waves are damped to various extent in rocks or soil.

It should be mentioned that as P, S or surface waves propagate through complicated rock strata, one type of wave may generate waves of another type. Also, phase shifts (cos δ) may occur as waves are reflected or refracted. In particular a vertically propagating horizontally polarized shear wave (SH) will double its amplitude when reflected from the free ground surface.

THE ELEMENTS OF COHERENCY

Estimation of Coherency

The strong motions (see Figure 2) at the various input points will, for the reasons already explained (such as source emission and path scattering), not be alike in general. A measure of the likeness of two wave trains is called the coherency and quantitative measures can be obtained in the time-domain as follows.

Consider Eq. (9). The factors $U_j(k)$ contain the exponential term which advances the phase of the wave harmonic component at station j with wave number k by the appropriate time delay.

The cross-spectral estimates are often normalized to unit amplitude to reduce site amplification effects. With this normalization, the cross-spectral matrix becomes simply the matrix of exponential phase differences

$$S_{jl}(\omega) = \sum_{m=-M}^{M} a_m \exp\{ i [\varphi(x_j, \frac{\omega+2\pi m}{T}) - \varphi(x_l, \frac{\omega+2\pi m}{T})]\}, \qquad (11)$$

where φ is the Fourier phase.

The simplest coherence measures are for two input points in which case the coherency is defined in the frequency domain by

$$\gamma_{12}(\omega) = S_{12}(\omega)/[S_{11}(\omega)S_{22}(\omega)]^{1/2}. \qquad (12)$$

The coherence is defined as the square of the modulus of the coherency and is a normalized measure, $0 \le |\gamma_{12}(\omega)|^2 \le 1$, where a value of 1 indicates complete coherence.

A number of coherency studies have now been published (see Abrahamson and Bolt, 1987) and a few have been incorporated into structural response analyses for critical structures such as the Diablo Canyon Nuclear Power Station as part of soil-structure interaction calculations. One example is given here in Figure 3 from strong motions recorded by the SMART 1 array (Abrahamson and others, 1987). At high frequencies (here above 2 Hz) the recorded motion (mainly the S waves) is dominated by incoherent energy when averaged over a large distance. A practical approach to modelling would be to generate a suite of synthetic ground motions in which the Fourier phase of the incoherent energy is varied in a statistical manner while the Fourier phase of the coherent energy remains deterministic.

CONSTRUCTION OF LAGGED TIME HISTORIES

Source and Site Specification

We are now in a position to outline the basic steps in synthesizing a set of time histories for inputs to a multi-supported structure. It should be stressed that this construction is not unique and that alternative procedures can be adopted at various stages. For example, computations can be performed purely in the time domain or in the spectral domain. At this early stage, each method has its proponents but in the long run the speed of computers will make feasible optimal construction by key cross-over exchanges between the two domains.

The first step is to define from geological and seismological information the appropriate earthquake sources for the site in question. The source specification may be deterministic or probabalistic in concept and may be decided on grounds of acceptable risk (Bolt, 1991) A necessary set of source parameters would be magnitude (moment magnitude), seismic moment, fault location and assumed rupture surface length and depth, fault strike and dip, faulting mechanism (strike-slip, normal, etc.), rupture velocity and rupture rise-time (both usually selected from an empirical regression curve). In fact, much of this latter detail is not essential for reasonably realistic synthesis of phased time histories.

On the other hand, it is essential to specify closely the propagation path distance, the P, S, and surface wave speeds in the vicinity of the site (particularly seismic velocities for the alluvial, deep soil and surficial rock layers). These speeds are needed to calculate the appropriate wave propagation delays between support points (see Eq. (4)) and the angles of approach of the incident waves. Some soil-structure interaction programs already permit calculations of this kind.

Strong Ground Motion Specification

The construction of realistic phased seismograms (Niazi, 1986) can be regarded as a series of iterations from the most appropriate observed strong motion record already available to a set of more specific time-histories which incorporate the physically defined phase patterns. Where feasible, a strong motion accelerogram is chosen which satisfies within allowable limits the seismic source and site specifications described above. Of course, for large near earthquake sources ($M > 7.5$), there are few or no actual recordings and a synthetic record must first be constructed by scaling and/or numerical modelling.

As the iteration proceeds, certain constraints on each member of the set of phased records must be satisfied. Thus all must have response amplitude spectra that fall within say one standard error of the target spectra. (The response phase spectra (see Figure 5) will, of course, vary within prescribed tolerances also although little research has as yet been done to define such limits.) Similarly, each member of the set must preserve pre-specified peak ground accelerations, velocities and displacements within statistical bounds. The durations of each section (mainly P, S and surface wave portions) of each time-history must also satisfy prescribed source, path, and site conditions.

In order to guide convergence of the iterative process described and to provide a check on the overall result, it is advantageous to have practical seismological advice on the product of the iterations. At the present, more or less automatic convergent algorithms are not available so that experience with observed seismograms and knowledge of the underlying seismic wave theory is especially valuable. In any event, it is important for the model methodology to be fully documented. Plastic overlays showing the phased time-histories and spectra on a standard scale are particularly valuable for checking.

ENGINEERING APPLICATIONS

Seismic Phase Response Spectrum

The effect of spatial variations of the phasing of strong ground motion on large structures where multi-support inputs are appropriate can be demonstrated using strong motion array data. We first need a way to incorporate the structural interaction.

The total response of the structure can be separated into the quasi-static response and the dynamic response. At a given node in the structure, the dynamic response due to the phase shifted inputs is divided by the mean response found using each of the individual support ground motions as rigid base inputs. This ratio, called the "dynamic response ratio", indicates the effect of the spatial variation of the ground motion on the dynamic response of the structure.

Consider a structure with N supports and M structural nodes and assume that the normal modes of the structure are known and that for each structural mode, the participation factor of the kth node to the lth input is also known. These weights, denoted w_{kl}, will depend on the mass and stiffness of the structure as well as the structural mode.

For simplicity, consider the special case of just one structural node with N inputs. The dynamic response of the structure satisfies the differential equation

$$\ddot{r}(t) + 2\xi\omega\dot{r}(t) + \omega^2 r(t) = -\vec{w} \cdot \ddot{u}(t), \tag{13}$$

where ξ is the damping, ω is the natural frequency, and $\ddot{u}(t)$ is the N length vector of support input accelerations.

Let $R_{\xi,T}(t)$ be the unit impulse response of a single degree of freedom oscillator with period T and damping ξ . The dynamic acceleration response is usually characterized by the maximum of $|\ddot{r}(t)|$. The acceleration response spectrum for Eq. (13) is denoted $SA(\xi, T)$ and is given by

$$SA(\xi,T) = \max_t \left\{ R_{\xi,T}(t) * \vec{w} \cdot \ddot{\vec{u}}(t) \right\}, \tag{14}$$

where * indicates a convolution. This response spectrum is compared to the response spectrum obtained by using rigid base inputs.

Using the lth input at all of the support nodes, the equation of motion becomes

$$\ddot{r}(t) + 2\xi\omega\dot{r}(t) + \omega^2 r(t) = \left(\sum_{k=1}^{N} w_k \right) \ddot{u}_l(t). \tag{15}$$

Let $SA_l(\xi, T)$ be the acceleration response spectrum for the ground acceleration $\ddot{u}_l(t)$. Then the mean response spectrum of all the inputs is given by

$$\overline{SA}(\xi,T) = \left(\sum_{k=1}^{N} w_k \right) \frac{1}{N} \sum_{l=1}^{N} SA_l(\xi,T). \tag{16}$$

The dynamic response ratio is given by

$$\Phi^d(\xi,T) = \frac{SA(\xi,T)}{\overline{SA}(\xi,T)}. \tag{17}$$

Substituting for the response spectra, Eq. (17) becomes

$$\Phi^d(\xi,T) = \frac{\max_t \left| R_{\xi,T}(t) * \vec{w} \cdot \ddot{\vec{u}}(t) \right|}{\left(\sum_{k=1}^{N} w_k \right) \frac{1}{N} \sum_{l=1}^{N} SA_l(\xi,T)}. \tag{18}$$

This ratio is influenced by both differential amplification of the ground motion due to site effects and by differential phasing of the ground motion. The differential phasing may be due to incoherent waves, nonvertically propagating waves or local site effects. To simplify the interpretation of the dynamic response ratio, site amplification effects are removed by normalizing each support acceleration $\ddot{u}_l(t)$ by the response $SA_l(\xi, T)$. The dynamic response ratio given by Eq. (18) has been evaluated using strong motion array recordings from SMART 1 as ground motion inputs at the supports. Descriptions of the SMART 1 array are given in Bolt et al. (1982). An illustration of the dynamic response ratio and response phase spectra from an earthquake recorded by SMART 1 are shown in Figures 4 and 5 (Abrahamson and Bolt, 1985).

San Francisco Bay Bridges, California

After the 1989 Loma Prieta earthquake, with the fall of the span on the Bay Bridge, much attention has been focussed on the appropriate design criteria for large structures in the vicinity of active faults. The Bay Area Rapid Transit Authority (BART) is in the design stage for extensions of the rail system, some of which pass across or near major active faults. The Golden Gate Bridge District has undertaken a detailed seismic analysis of the Golden Gate Bridge, which might be shaken by an earthquake resembling that in 1906 on the adjacent San Andreas fault. As well, Cal Trans has initiated ground motion studies for its existing and future bridges both in the San Francisco Bay area and in the Los Angeles area.

In all the above cases, analysis must be focussed on relatively long period seismic motions, i.e. from 2 Hz to 5 sec or more. It should be remembered that for wave components (see Eq. (1)) of period 1 sec the S-wave lengths λ incident on the supports are from several hundred to a thousand meters, or of the same order as key bridge dimensions. For this reason, specifications for the studies have called for the consideration of the effects of phased strong-motion inputs and, in some cases, for the incorporation of coherency factors. These studies present a considerable challenge because of the lack of suitable observational material. Few strong motion records are available for large near-earthquakes that measure spatial variation of shaking over distances of hundreds of meters. Also, because of the size of the engineered structures and the relatively large distances between the support points, ground velocity and displacement become of critical importance. The reliability of ground displacement records obtained from accelerograms is a question that needs additional checking.

ACKNOWLEDGEMENTS

A number of examples discussed in the text were worked in collaboration with Dr. N. A. Abrahamson and Dr. J. P. Singh, to whom I extend my thanks. Mr. N. Gregor helped with text preparation. The research was supported by NSF CES88-00457.

REFERENCES

Abrahamson, N. A., B. A. Bolt, R. B. Darragh, J. Penzien and Y. B. Tsai, 1987. The SMART 1 accelerograph array (1980-1987): A review. Earthquake Spectra, 3, 263-284.

Abrahamson, N. A. and B. A. Bolt, 1985. The spatial variation of the phasing of seismic strong ground motion. Bull. Seism. Soc. Am., 75, 1247-1264.

Abrahamson, N. A. and B. A. Bolt, 1987. Array analysis and synthesis mapping of strong seismic motion. In: B. A. Bolt (editor), Strong motion synthetics: Computational techniques series. Academic Press, New York.

Bolt, B. A., 1991. Balance of risks and benefits in preparation for earthquakes. Science, 251, 169-174.
Bolt, B. A., C. H. Loh, J. Penzien, Y. B. Tsai and Y. T. Yeh 1982. Preliminary report on SMART 1 strong motion array in Taiwan. EERC Report No. UCB/EERC-82/13.
Bullen, K. E. and B. A. Bolt, 1985. An introduction to the theory of seismology. Cambridge University Press, New York.
Harichandran, S. and E. H. Vanmarcke, 1986. Stochastic variation of earthquake ground motion in space and time. J. Engin. Mech., 112, 154-174.
Luco, J. E. and H. L. Wong, 1986. Response of a rigid foundation to a spatially random ground motion. Earth. Engin. Struct. Dyn., 14, 891-908.
Niazi, M., 1986. Inferred displacements, velocities and rotations along a long rigid foundation located at El Centro Differential Array site during the 1979 Imperial Valley, California, earthquake. Earth. Engin. Struct. Dyn., 14, 531-542.

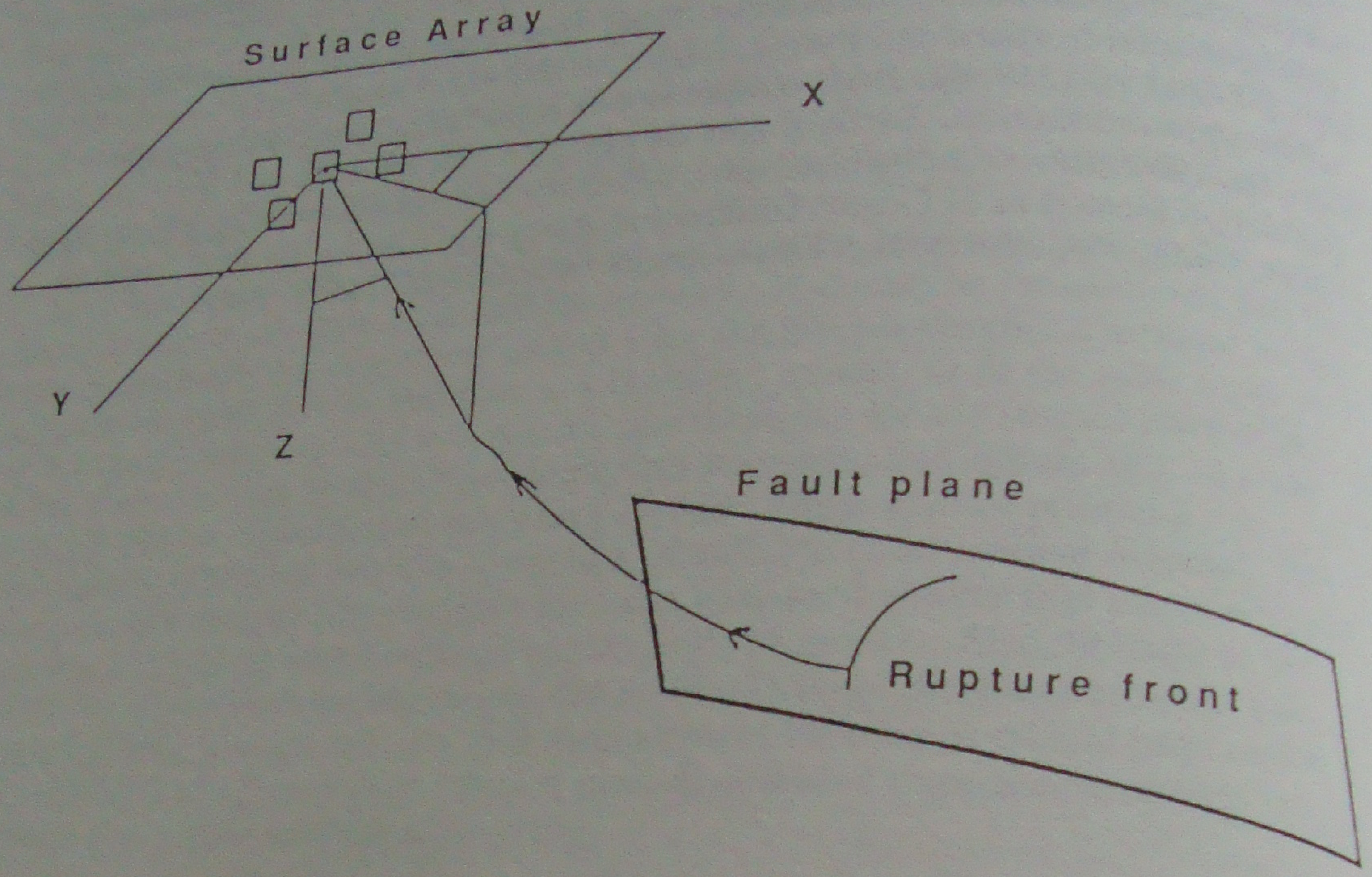

Fig. 1 Passage of a seismic wave from the seismic source to multi-stations in an array or to multi-supports.

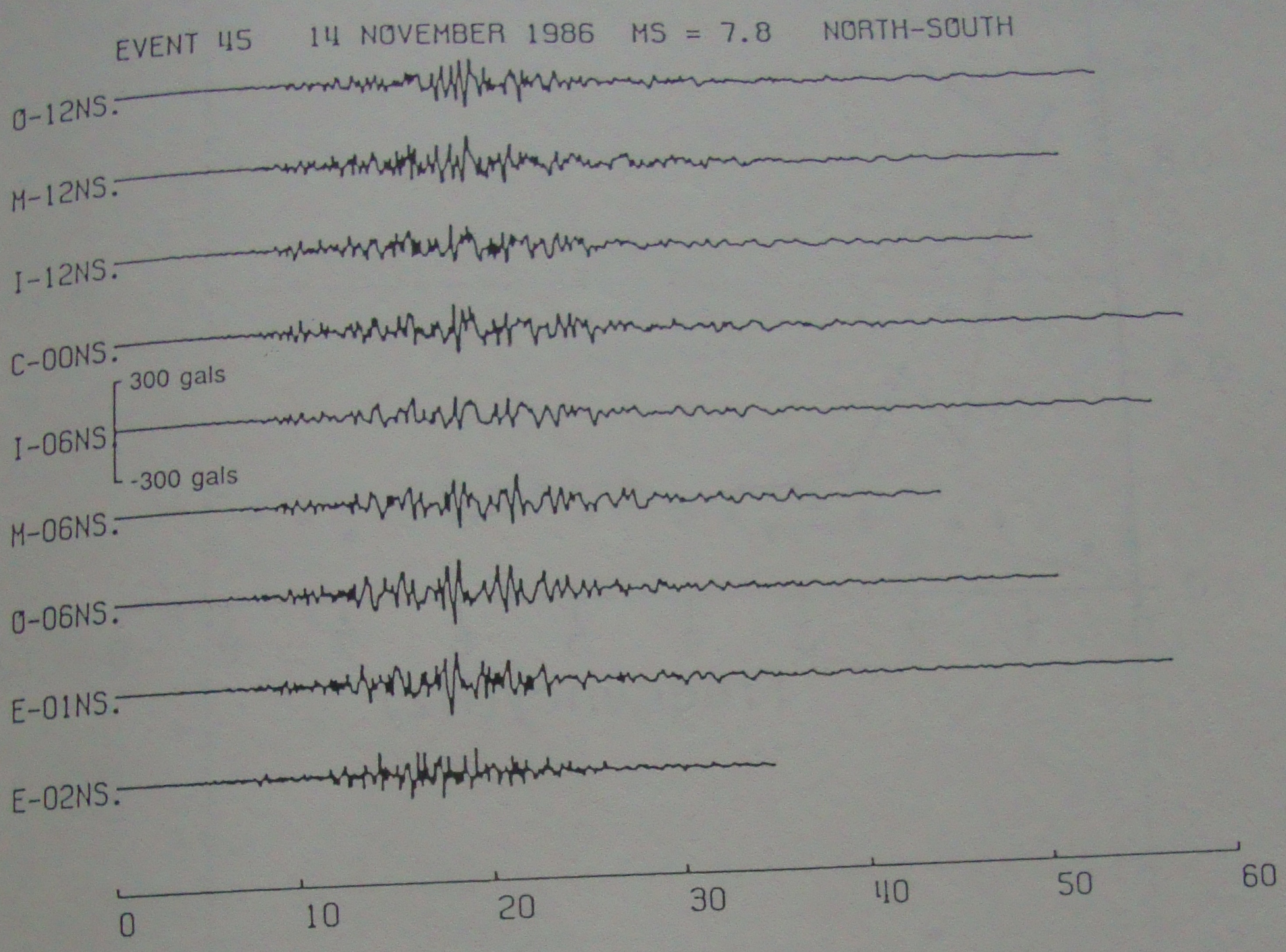

Fig. 2 Differential phasing in the horizontal ground acceleration across the SMART 1 array in Taiwan. The horizontal scale is in seconds. The distance between the central station C00 and the inner ring (I) is 200 m and the outer ring (O) is 1000 m.

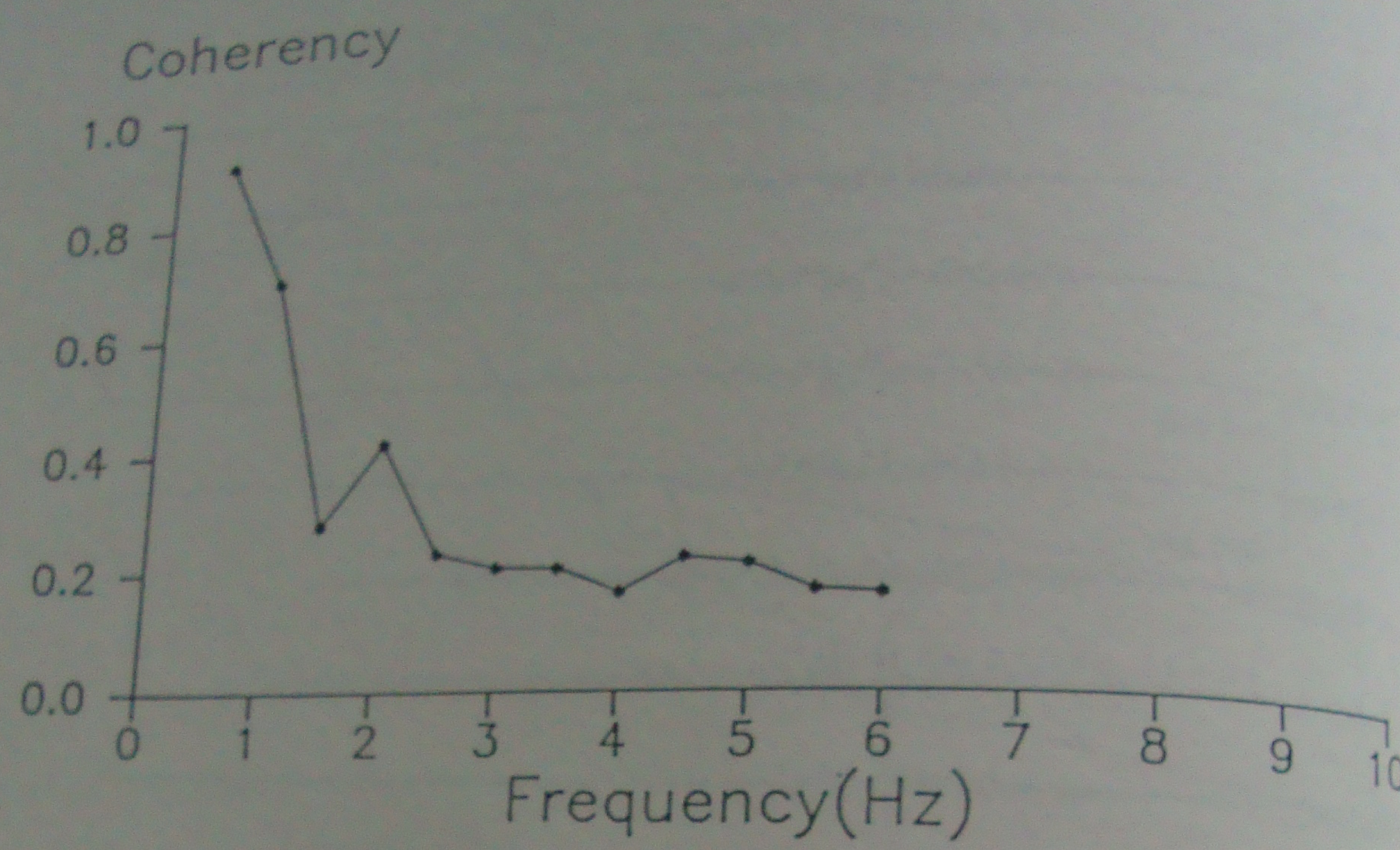

Fig. 3 Calculated coherency values of earthquake number 43, north-south component for all 37 stations of the SMART 1 array. The time window used was from 5 to 10 sec. along the record.

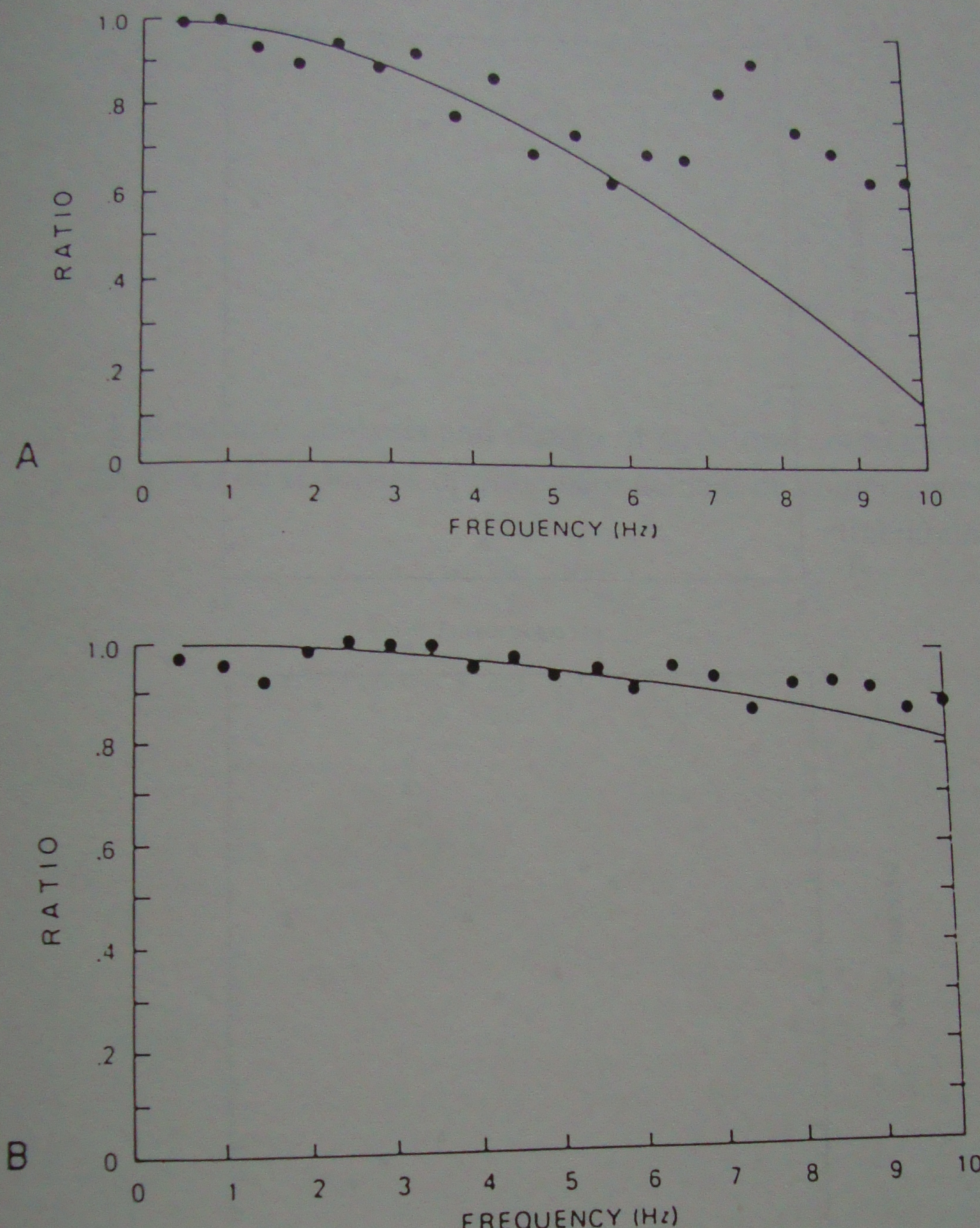

Fig. 4 The fundamental mode (in-phase motion) dynamic response ratio for 200-m support spacing using the tranverse component of acceleration from event 5. The filled circles are the estimated ratios, and the line gives the ration expected for a simple plane wave propagating from the source region. (A) station pair C00-I06. (B) Station pair C00-I03.

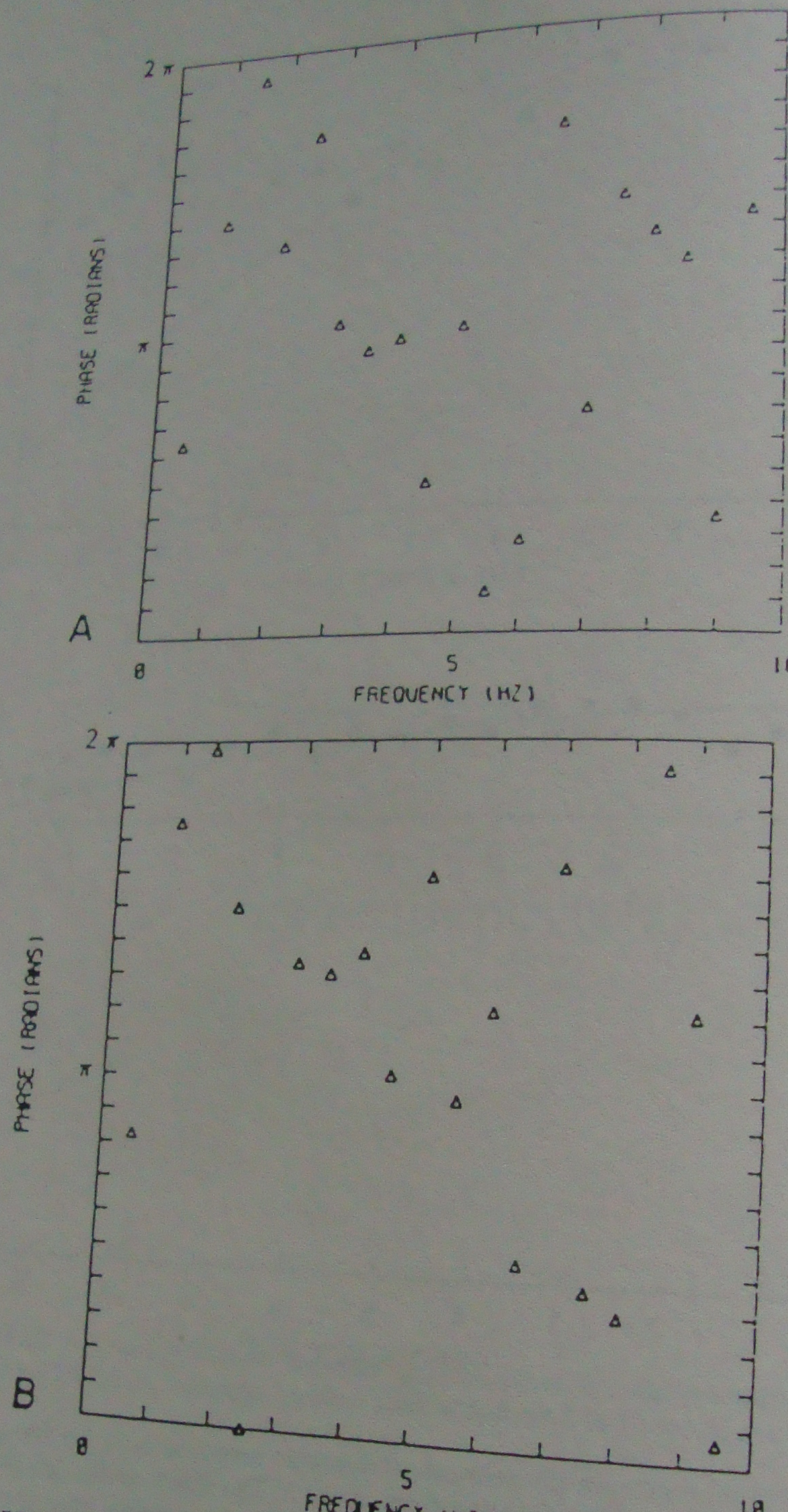

Fig. 5 (A) The response phase spectra $\Psi_{\xi}^{\kappa}(\omega)$ at 5 per cent damping. Station C00, transverse component. (B) Station I06, transverse component.